Question 1

(a) (1) $|2x-3| \leq 0.4$

(2) $0 < |x-5| < 0.5$.

(1) $|2x-3| \leq 0.4$.

$-0.4 \leq 2x-3 \leq 0.4$.

$2x-3 \geq -0.4$

$2x \geq -0.4+3 = 2.6$

$x \geq 1.3$

$2x-3 \leq 0.4$.

$2x \leq 0.4+3 = 3.4$

$x \leq 1.7$

$(1.3 \leq x \leq 1.7)$

(2) $0 < |x-5| < 0.5$.

$0 < x-5$ $\qquad$ $x-5 < 0.5$

$\cancel{x}\ +5 < x$. $\qquad$ $x < 0.5+5 = 5.5$.

$5 < x < 5.5$

b) (3) $|x+3| = |2x+1|$.

$x+3 = 2x+1$

$x-2x = 1-3$.

$-x = -2$

$\underline{\underline{x = 2}}$.

(4) $\left|\frac{2x-1}{x+1}\right| = 3$.

$\frac{2x-1}{x+1} = 3$

$2x-1 = 3(x+1)$

$2x-1 = 3x+3$

$2x-3x = 3+1$

$-x = 4$

$\underline{\underline{x = -4}}$. $= \underline{\underline{-4}}$.

(c) (5) $x^3+4x < 4x^2$

$$x^3+4x-4x^2 < 0.$$

$$x(x^2-4x+4) < 0.$$

$$x(x^2-4x+4) < 0$$

$$x(x(x-4)-1(x-4) < 0$$

$$x((x-1)(x-4)) < 0.$$

$x(x-1)(x-4) < 0$ has the solutions 0, 1, and 4

The numbers 0, 1 and 4 divide the real line in four intervals.

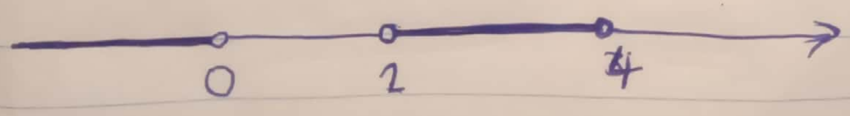
$(-\infty,0)$, $(0,1)$, $(1,3)$ and $(3,\infty)$.

Sign of the factors chart is;

Interval	x	$x-1$	$x-4$	$x(x-1)(x-4)$
$x<0$	$-$	$-$	$-$	$-$
$0<x<1$	$+$	$-$	$-$	$+$
$1<x<4$	$+$	$+$	$-$	$-$
$x>4$	$+$	$+$	$+$	$+$

From the chart $x^3+4x<4x^2$ is negative when $x<0$, and $1<x<4$.

Thus, the solution of the inequality is the interval $(-\infty,0)\cup(1,4)$.



Question 2

(a) (1) $\frac{-2^6}{4^3} = \frac{-2^6}{2^{2(3)}} = \frac{-2^6}{2^6} = -2^{(6-6)} = -2^0$

$= -2^0 = \cancel{0}\ -1$

(2) $\frac{x^3 x^n}{x^{n+1}} = \frac{x^{3+n}}{x^{n+1}} = x^{3+n+n+1} = x^{4+2n}$

$= x^{4+2n}$

(3) $\dfrac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}} = \dfrac{(a\sqrt{b})^{\frac{1}{2}}}{\sqrt[3]{ab}} = \cancel{a\sqrt{b}}$

$= \dfrac{(a\sqrt{b})^{\frac{1}{2}}}{(ab)^{\frac{1}{3}}} = \dfrac{a^{\frac{1}{2}}(\sqrt{b})^{\frac{1}{2}}}{a^{\frac{1}{3}}b^{\frac{1}{3}}}$

$\dfrac{a^{\frac{1}{2}}}{a^{\frac{1}{3}}} = a^{\frac{1}{2}-\frac{1}{3}} = a^{\frac{1}{6}}$

$= \dfrac{a^{\frac{1}{6}}(\sqrt{b})^{\frac{1}{2}}}{b^{\frac{1}{3}}}$, $\dfrac{(\sqrt{b})^{\frac{1}{2}}}{b^{\frac{1}{3}}} = \dfrac{b^{\frac{1}{4}}}{b^{\frac{1}{3}}} = b^{\frac{1}{4}-\frac{1}{3}} = -\frac{1}{12}$

$= b^{-\frac{1}{12}}$.

$= a^{\frac{1}{6}}\, b^{-\frac{1}{12}} = \dfrac{\sqrt[6]{a}}{\sqrt[12]{b}}$

(b) (4) $f(x) = \frac{1 - e^{x^2}}{1 - e^{1-x^2}}$

$1 - e^{1-x^2} \neq 0.$

$e^{1-x^2} \neq 1.$

$x \neq -1, 1.$

Domain is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

(5) $f(x) = \frac{1+x}{e^{\cos x}}$

$e^{\cos x}$ is always positive

Domain is $(-\infty, \infty)$.

Question 3.

(a) $\log_3\left(\frac{1}{81}\right) = \log_3(1) - \log_3 81.$

$\log_3(1) = x$

$1 = 3^x.$

$\log 1 = \log 3^x = x\log 3.$

$x = \frac{\log 1}{\log 3} = 0.$

$\log_3 81 = y.$

$81 = 3^y.$

$\log 81 = \log 3^y$

$\log 3^4 = y\log 3. = 4\log 3.$

$y = \frac{4\log 3}{\log 3} = 4.$

$\log_3\left(\frac{1}{81}\right) = 0 - 4 = \underline{\underline{-4}}.$

(b) $\cancel{e^{-2}\ln 5}$. $e^{-2\ln 5}$

Say $e^{-2\ln 5} = L.$

$\ln L = [\ln 5(\ln e)]^{-2}.$

$\ln e = 1$

$\log L = (5)^{-2}$

$L = \frac{1}{25}$

0. 36

(c) $e\ln(\ln e^3)$.

let $e\ln(\ln e^3) = y$.

$\ln e \cdot \ln(3\ln e) = \ln y$.

$\ln e = 1$.

$1 \cdot \ln(3(1)) = \ln y$.

$\ln 3 = \ln y$

$y = 3$.

$= \underline{\underline{3}}$

(d) $\ln \frac{1}{e^2}$

let $\ln \frac{1}{e^2} = x$.

$\ln e^{-2} = x$.

$-2\ln e = x$

$\ln e = 1$.

$\underline{\underline{x = -2}}$

(e) $2\log_5 100 - 4\log_5 50$.

$\log_5 100^2 - \log_5 50^4$

$= \log_5\left(\frac{100^2}{50^4}\right) = \log_5\left(\frac{1}{625}\right)$.

let

$\log_5\left(\frac{1}{625}\right) = x$.

$\frac{1}{625} = 5^x$.

$\log\left(\frac{1}{625}\right) = \log 5^x$.

$-\log(5^4) = x\log 5$.

$x = \frac{-4\log 5}{\log 5} = \underline{\underline{-4}}$.

Question 6

6. (1) $\cos\left(\frac{\pi}{2} - x\right) = \sin x$.

$\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$.

$\cos\frac{\pi}{2} = \cos(90) = 0$.

$\sin\frac{\pi}{2} = \sin(90) = 1$.

$= 0(\cos x) + 1(\sin x)$.

$= 0 + \sin x$

$= \sin x$. Hence the prove.

(2) $(\sin x + \cos x)^2 = 1 + \sin 2x$.

$(\sin x + \cos x)^2 = (\sin x + \cos x)(\sin x + \cos x)$.

$= \sin^2 x + \sin x \cos x + \cos^2 x + \sin x \cos x$.

$= \sin^2 x + \cos^2 x + 2\sin x \cos x$.

but $(\sin^2 x + \cos^2 x) = 1$.

$= 1 + 2\sin x \cos x$.

but

$$2\sin x \cos x = \sin 2x .$$

$$= 1 + \sin 2x .$$

(2) $\tan^2\alpha - \sin^2\alpha = \tan^2\alpha \sin^2\alpha$.

$\tan^2 x \sin^2 x = (1 - \cos^2(x))\tan^2(x)$.

$1 - \cos^2(x))(\tan^2(x)) = \tan^2 x - \tan^2 x \cos^2 x$.

$\tan\theta = \frac{\sin\theta}{\tan\theta}$

but $-\cos^2(x)\tan^2(x) = -\sin^2 x$.

$= \tan^2 x - \sin^2 x$.

(4) $2\csc 2t = \sec t \csc t$.

$$2\csc 2t = \frac{2}{\sin(2t)} = \frac{1}{\cos(t)\sin(t)}.$$

$$= \frac{1}{\cos(t)} \cdot \frac{1}{\sin(t)} =$$

$\sin t = \frac{1}{\csc t}$, $\cos(t) = \frac{1}{\sec t}$.

$$= \frac{1}{\cos(t)\sin(t)} = \underline{\underline{\csc t \sec t}}.$$

= Hence Proven.